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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2018

FIRST YEAR [BATCH 2017-20]

Date : 24/05/2018 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper : II** Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer any three questions from Question Nos. 1 to 5 : [3×5]

1. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two parallel straight lines, then show that distance between them is $2\sqrt{\dfrac{g^2-ac}{a(a+b)}}$. [5]

2. a) For what value of λ does $\lambda xy-8x+9y-12=0$ represent a pair of lines? [2]
   b) Prove that $bx^2-2hxy+ay^2=0$ represents two straight lines at right angles to the lines represented by $ax^2+2hxy+by^2=0$. [3]

3. Show that the locus of the poles of chords of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ which subtend a right angle at the centre is $\dfrac{x^2}{a^4}+\dfrac{y^2}{b^4}=\dfrac{1}{a^2}-\dfrac{1}{b^2}$. [5]

4. Two tangents drawn to the parabola $y^2=4ax$ meet at angle 45°. Show that the locus of their point of intersection is $(x+a)^2=y^2-4ax$. [5]

5. Reducing the equation $4x^2+4xy+y^2-4x-2y+a=0$ to its canonical form, determine the nature of the conic for different values of a. [3+2]

Answer any three questions from Question Nos. 6 to 10 : [3×5]

6. Show that the shortest distance between the straight lines $\vec{r}=\vec{a}+t\vec{\alpha}$ and $\vec{r}=\vec{b}+s\vec{\beta}$, where $\vec{a}=(6\hat{i}+2\hat{j}+2\hat{k})$, $\vec{b}=(-4\hat{i}-\hat{k})$, $\vec{\alpha}=(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{\beta}=(3\hat{i}-2\hat{j}-2\hat{k})$ is 9 units. [5]

7. Given two vectors $\vec{\alpha}=3\hat{i}-\hat{j}$ and $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$. Express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. [5]

8. Show that the equation of a plane which contains the straight line $\vec{r}=\vec{a}+t\vec{b}$ and is perpendicular to the plane $\vec{r}\cdot\vec{\delta}=q$ is $[\vec{r}-\vec{a}\quad \vec{b}\quad \vec{\delta}]=0$. [5]

9. A particle acted on by a force of 15 units, is displaced from the point (1, 1, 1) to the point (2, 1, 3). If the line of action of the force be the vector $(\hat{i}+2\hat{j}+2\hat{k})$, then show that the work-done by the force is 25 units of work. [5]

10. Show that in a triangle the perpendiculars drawn from the vertices to the opposite sides are concurrent. [5]

Group - B

***Answer any five questions from Question Nos. 11 to 18* :** [5×5]

11. Examine the convergence of the series : $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...+\frac{1}{(n-1)!}+....$

12. Define Cauchy's general principle of convergence of a sequence. Prove or disprove that the sequence $\left\{\frac{1}{n}\right\}$ is a Cauchy sequence. [2+3]

13. If $f(h)=f(0)+hf'(0)+\frac{h^2}{2!}f''(\theta h)$, $0<\theta<1$, find θ, when $h=1$ and $f(x)=(1-x)^{\frac{5}{2}}$.

14. Examine the validity of the Mean Value Theorem for $f(x)=4-(6-x)^{\frac{2}{3}}$ in [5,7].

15. Show that of all rectangles of given area, the square has the smallest perimeter.

16. Determine the asymptotes of $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$.

17. Find the envelope of the straight lines $\frac{x}{a}+\frac{y}{b}=1$, where a and b are parameters, connected by $a+b=c$, c being a non zero constant.

18. Find a, if $\lim\limits_{x\to 0}\frac{e^x-ae^{x\cos x}}{x-\sin x}$ is finite. What is the value of the limit then? [3+2]

***Answer any two questions from Question Nos. 19 to 21* :** [2×5]

19. Evaluate : $\int\frac{\cos x+2\sin x}{3\cos x+4\sin x}dx$ [5]

20. a) Evaluate the following : $\lim\limits_{n\to\infty}\left[\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+...+\frac{n^2}{2n^3}\right]$ [3]

    b) Show that, $\int_0^{na}f(x)dx=n\int_0^a f(x)dx$ if $f(x)=f(a+x)$ [2]

21. If $J_n=\int\limits_0^{\frac{\pi}{2}}x^n\sin x\,dx$ (n > 1, an integer), show that $J_n+n(n-1)J_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find the value of $\int\limits_0^{\frac{\pi}{2}}x^5\sin x\,dx$. [3+2]

***Answer any two questions from Question Nos. 22 to 24* :** [2×5]

22. Reduce the differential equation $\sin y\frac{dy}{dx}=\cos x(2\cos y-\sin^2 x)$ to a linear equation and hence solve it. [2+3]

23. Reduce the differential equation $xp^2-2yp+x+2y=0$ $\left(p=\frac{dy}{dx}\right)$ to Clairaut's form by transforming $x^2=u$ and $y-x=v$. Find its general solution. [4+1]

24. Solve : $\frac{dy}{dx}=\frac{x+2y-3}{2x+y-3}$. [5]

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